Overload-based cascades on multiplex networks

Dong Zhou[1], Ahmed Elmokashfi[1]

[1] Simula Research Laboratory, Lysaker, Norway

ABSTRACT

Although cascading failures caused by overload in interdependent/interconnected networks have been studied in the recent years, the interplay between the extent of overlapping links (inter-similarity) and overload cascades is not well understood. This is an important issue since shared links do exist in many real-world coupled transportation systems. We propose a new model for load-based cascading failures on multiplex networks (based on previous single-layer models [1]). This model compares different network structures, coupling schemes and overload rules. More importantly, we systematically investigate the effects of inter-similarity on the robustness of the whole system under an initial intentional attack. Surprisingly, we find that inter-similarity sometimes can have both positive and negative impacts on multiplex traffic systems. It not only brings assortativity, but also causes more overloads to the same node on both layers. This also results in continuous changes of the best overlap ratio from 1 to 0 during the transition process of the system. These results provide useful suggestions for designing robust coupled traffic systems.

MODEL DESCRIPTION

- **Multiplex networks with loads**
  - Two subnetworks A, B with size N
  - Loads on each network: $L_i^A(t)$, $L_i^B(t)$, defined by betweenness centrality
  - Capacities on each network: $C_i^A = (1+\alpha_A)\cdot L_i^A(0)$
    $C_i^B = (1+\alpha_B)\cdot L_i^B(0)$
- **Overload cascades**
  - **Initial attack**: remove the node with largest total load $L_i^A(0) + L_i^B(0)$
  - Three overload rules: remove node i if
    - **OR rule**: $L_i^A(t) > C_i^A$ or $L_i^B(t) > C_i^B$
    - **SUM rule**: $L_i^A(t) + L_i^B(t) > C_i^A + C_i^B$
    - **AND rule**: $L_i^A(t) > C_i^A$ and $L_i^B(t) > C_i^B$
  - System robustness measure: relative mutual giant component size G
- **Coupling schemes**
  - Random coupling with a fraction r of overlapping links (for multiplex ERs or SFs)
  - Random/assortative/disassortative coupling

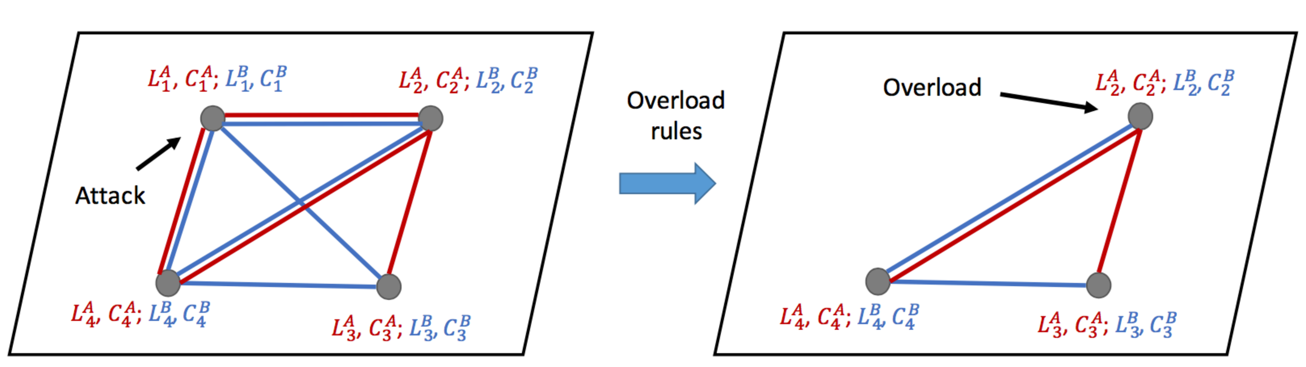

Fig. 1. **Model schematic.**

EFFECTS OF INTER-SIMILARITY

- **Two ER networks**
  - OR/AND rules: $r_{\text{best}} = 1$ and $r_{\text{best}} = 0$
  - SUM rule: r_{best} changes from 1 to 0 abruptly, but $r_{\text{best}} = 1$ during the transition
- **Two SF networks**
  - OR rule: $r_{\text{best}} = 1$ for most symmetric systems
  - SUM rule: continuous changes of r_{best} from 1 to 0 during the transition
  - AND rule: $r_{\text{best}} = 0$

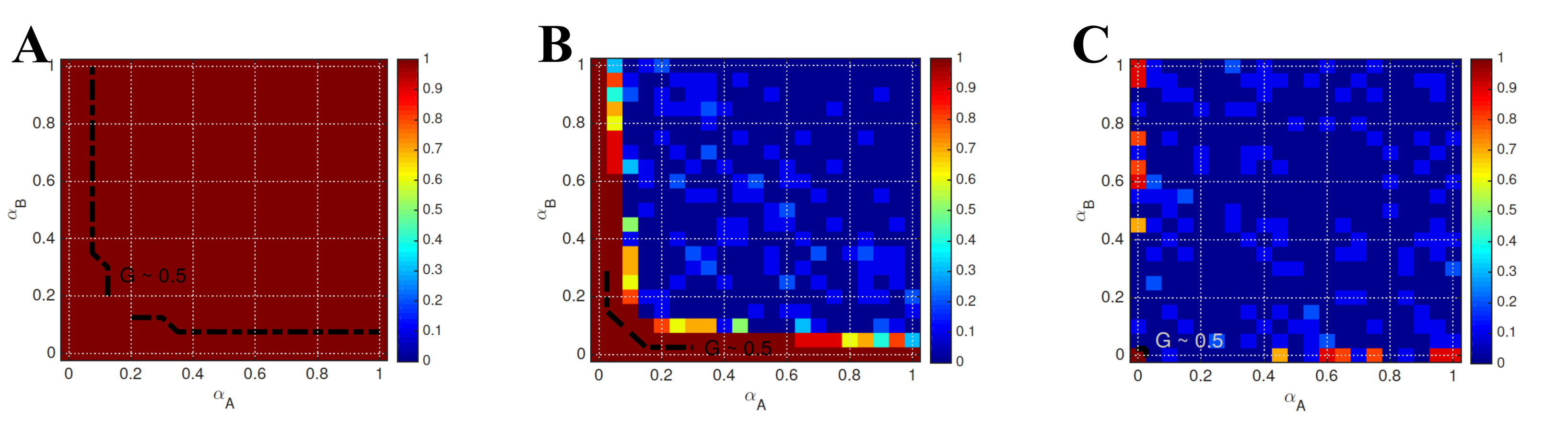

Fig. 2. **Best choice of overlap ratio for two ER networks.** (a) OR rule. (b) SUM rule. (c) AND rule. $N = 500$, $k_A = k_B = 6$, averaged over $M = 90$ realizations.

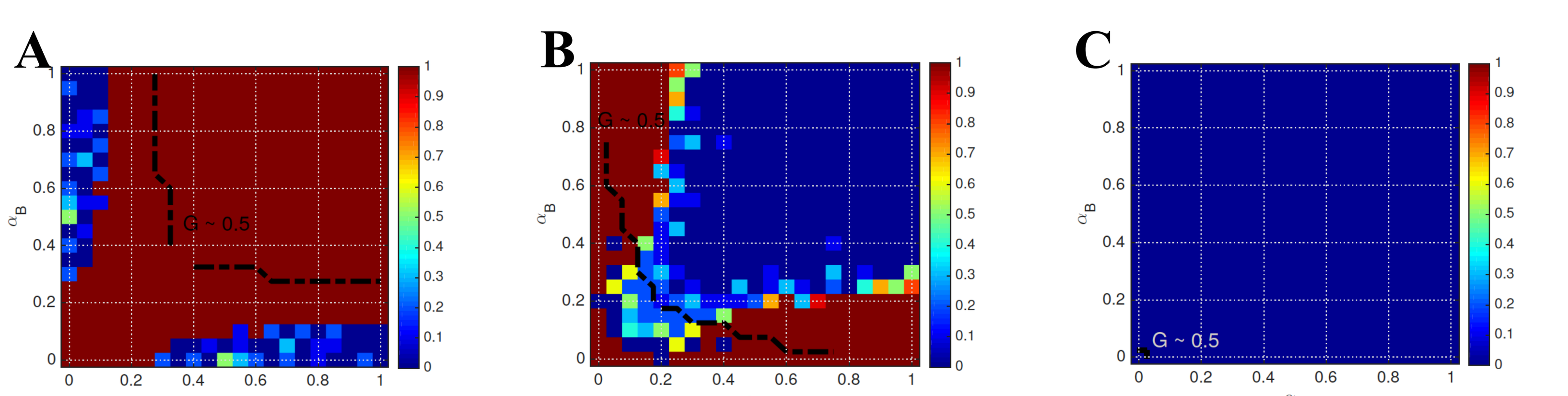

Fig. 3. **Best choice of overlap ratio for two SF networks.** (a) OR rule. (b) SUM rule. (c) AND rule. $N = 500$, $k_A = k_B = 6$, averaged over $M = 90$ realizations.

BEST CHOICE FOR THE SYSTEM

- **Two ER networks**
  - OR rule: "100% overlap" is the best in most cases
  - SUM rule: "100% overlap" -> "assortative coupling" -> "disassortative coupling";
    "assortative coupling" can be the best when $r_{\text{best}} = 1$
  - AND rule: "disassortative coupling" is the best in most cases;
    "assortative coupling" can be the best when $r_{\text{best}} = 1$ during the transition
- **Two SF networks**
  - OR rule: "100% overlap" is the best for most symmetric systems
  - SUM rule: "50% overlap" is the best during the transition (when assortativity is good);
    "assortative coupling" can be the best when $r_{\text{best}} = 1$ (non-symmetric)
  - AND rule: "disassortative coupling" is the best in most cases

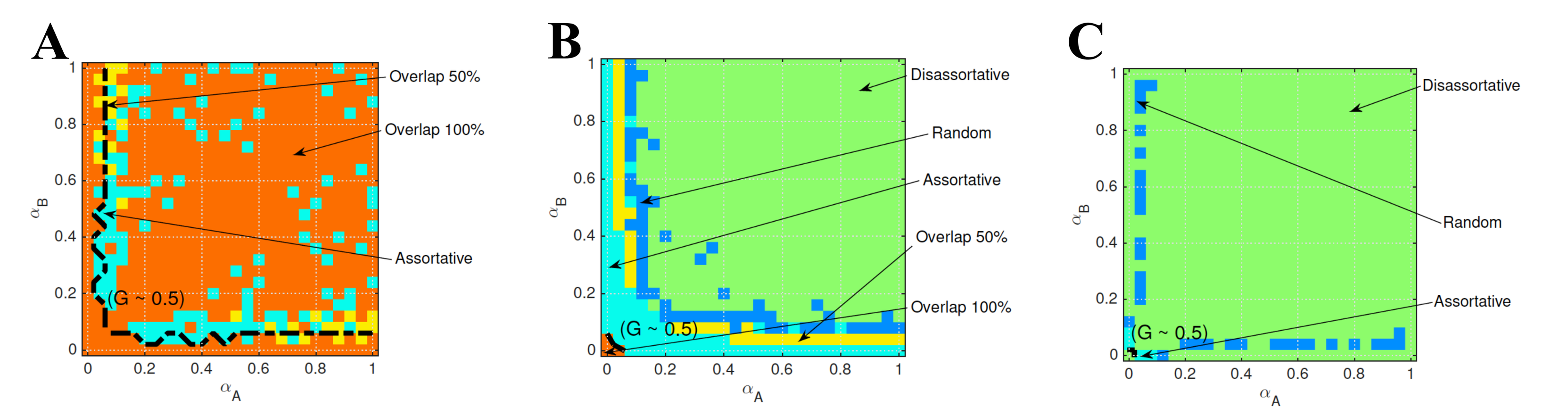

Fig. 4. **Best choice of coupling schemes for two ER networks.** (a) OR rule. (b) SUM rule. (c) AND rule. $N = 500$, $k_A = k_B = 6$, averaged over $M = 50$ realizations.

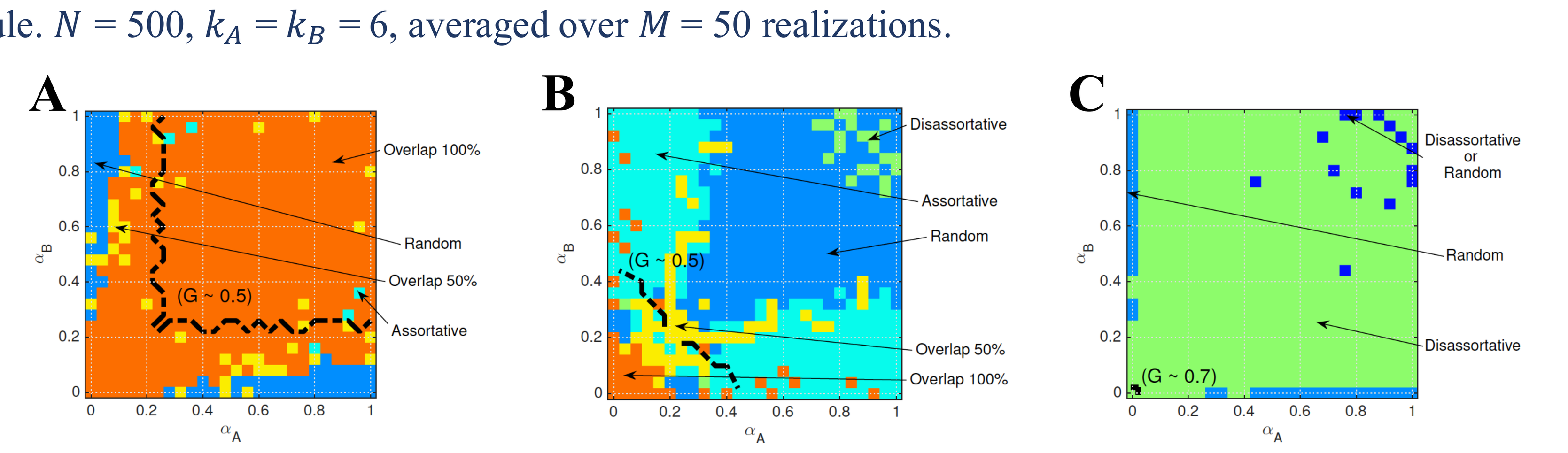

Fig. 5. **Best choice of coupling schemes for two SF networks.** (a) OR rule. (b) SUM rule. (c) AND rule. $N = 500$, $k_A = k_B = 6$, averaged over $M = 50$ realizations.

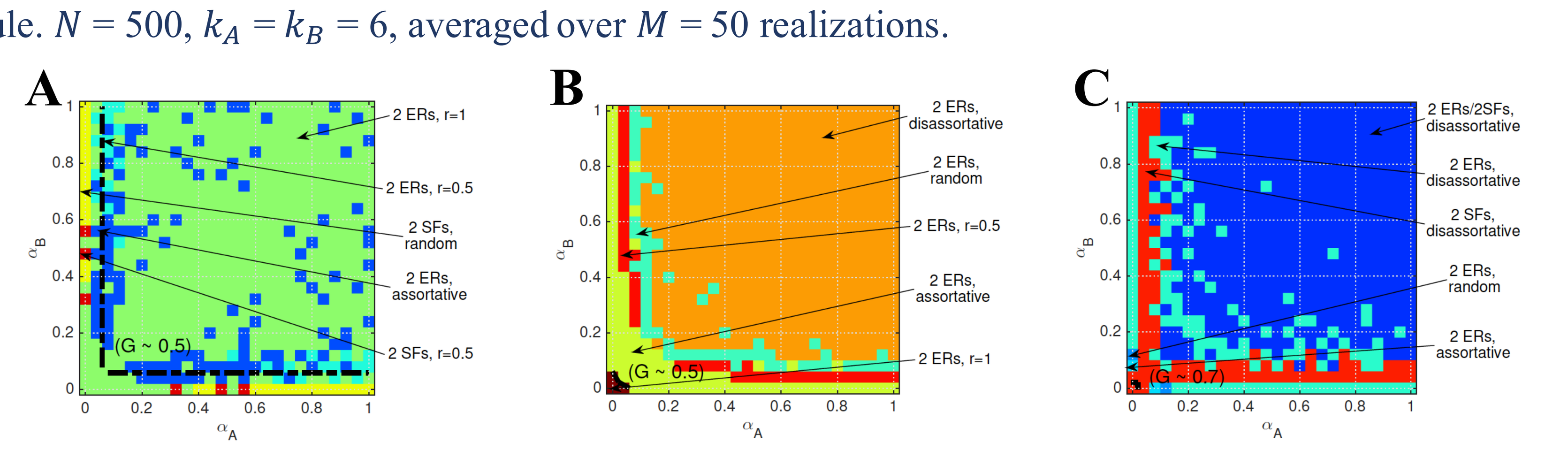

Fig. 6. **Best choice of network structures and coupling schemes.** (a) OR rule. (b) SUM rule. (c) AND rule. $N = 500$, $k_A = k_B = 6$, averaged over $M = 50$ realizations.

SUMMARY

- **Main findings**
  - Two SFs + SUM rule:
    r_{best} changes continuously during the transition, where assortativity is good
  - Two ERs + SUM/AND rule (or two SFs + SUM rule with non-symmetric systems):
    "100% overlap" can be the best ratio, but worse than assortative coupling
  - Two ERs are better than two SFs in most cases
- **Explanations & conclusions**
  - Inter-similarity has both positive and negative effects on multiplex traffic systems
    - **Positive effects**: introducing assortativity (when assortative coupling is superior to random/disassortative coupling)
    - **Negative effects** (for SUM/AND rule): enlarging the probability of having overloads for the same node on both layers
  - Homogeneous networks perform better than heterogeneous networks
  - Non-symmetric and symmetric systems can have different coupling preferences

ACKNOWLEDGMENT

This work is supported by Norwegian Research Council through DOMINOS project.

[1] Motter, A. E. & Lai, Y.-C. Cascade-based attacks on complex networks. Phys. Rev. E 66, 65102 (2002).

Contact: dong@simula.no