The Lattice Boltzmann Method and Turbidity Flow Modeling

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Overview

- Physical problem

- Conventional approach

$$\frac{\partial h}{\partial t} + \frac{\partial Uh}{\partial s} = (1-\delta)e_w U - \delta v_s$$

$$\frac{\partial Ch}{\partial t} + \frac{\partial UCh}{\partial s} = -v_s C$$

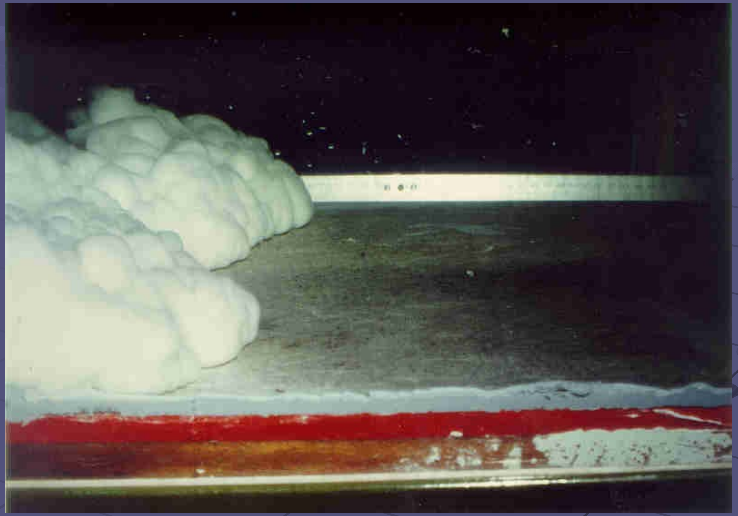
$$\frac{\partial Uh}{\partial t} + \frac{\partial U^2 h}{\partial s} + \delta U v_s = -\frac{1}{2} Rg \frac{\partial Ch^2}{\partial x} + RgChS - C_{f0} U^2$$

$$(1-\lambda_p)\frac{\partial \eta}{\partial t} = -v_s C$$

- Particle-based methods: Lattice Boltzmann approach

Physical problem

- Turbidity currents often arise from underwater avalanches
- Triggered by tsunamis and earthquakes
- Sand and water mix to create turbulent fluid
- Very complex fluid flow problem

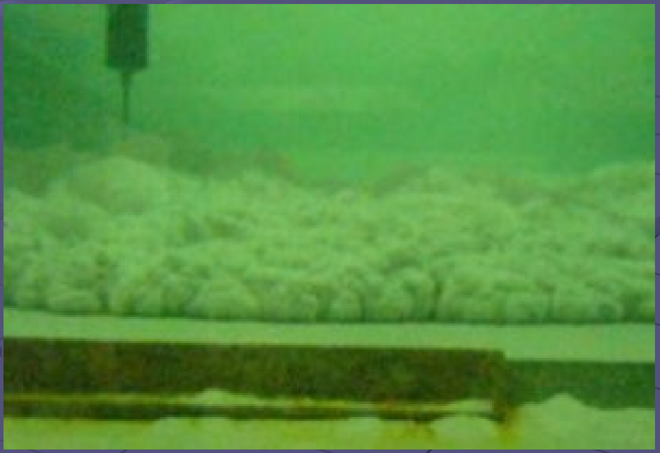


Physical Problem cont.

- Turbidity currents settle into turbidites
- Turbidites form a basis for hydrocarbon reservoirs
- Very challenging physical problem

Conventional Computational Fluid Dynamics

- Averaged multiphase fluid approach
- Difficult to construct appropriate constitutive relations
- Requires fitting of many, often non-physical, parameters



$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(uh) = E_w u \qquad (1)$$

$$\frac{\partial}{\partial t}(uh) + \frac{\partial}{\partial x}\left(u^2 h\right) = -\frac{(\rho_s - \rho_w)g}{2\rho_w}\frac{\partial}{\partial x}\left(Ch^2\right) + \frac{(\rho_s - \rho_w)ghCS}{\rho_w} - C_d(1+\alpha)u^2 \qquad (2)$$

$$\frac{\partial}{\partial t}(Ch) + \frac{\partial}{\partial x}(uCh) = -F_d + F_e \qquad (3)$$

Particle-based Methods

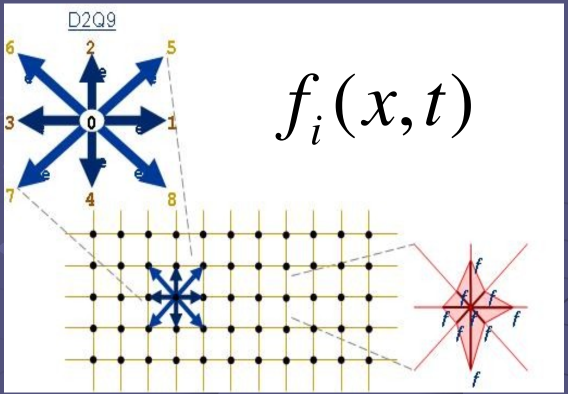
- Different philosophy than conventional computational fluid dynamics
- Discrete set of interacting particles act as foundation
- Usually simple rules that describe particle interaction

Lattice Boltzmann Method

- A quasi-particle description of the fluid
- The microscopic particle dynamics is averaged to obtain a simplified algorithm
- In principle, a discrete version of the Boltzmann equation on a regular lattice

$$\frac{df}{dt} = C[f]$$

Basic Lattice Boltzmann Theory



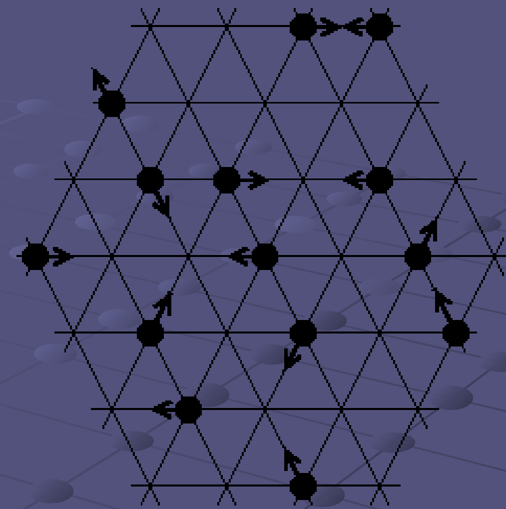
- Fluid described by a distribution function
- Macroscopic fluid quantaties are derived from mesoscopic quantaties
- The Lattice Boltzmann equation is

$$f_i(\mathbf{x}+c_i, t+1) = f_i(\mathbf{x}, t) + \frac{f_i(\mathbf{x}, t) - f_i^{eq}(\mathbf{x}, t)}{\tau}$$

- The equilibrium distribution function

$$f_i^{eq}(x,t) = w_i \rho \quad 1 + 3c_i \; u + \frac{9}{2}(c_i \; u)^2 - \frac{3}{2}u^2$$

Lattice Boltzmann Algorithm



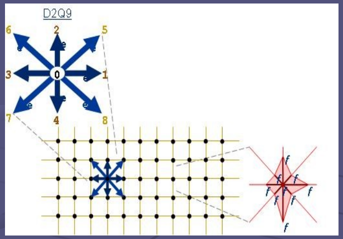
- Very similar to an explicit finite difference scheme
- Algorithm consits of two basic steps, set up to conserve mass and momentum
- Collision :

$$\tilde{f}_i(\vec{x},t) = f_i(\vec{x},t) - \frac{1}{\tau}\left(f_i(\vec{x},t) - f_i^{eq}(\vec{x},t) \right)$$

- Streaming :

$$f_i(x + c_i, t+1) = \tilde{f}_i(x,t)$$

Lattice Boltzmann Algorithm cont.



- At each time step, macroscopic quantaties are calculated from the distribution function

$$\rho(x,t) = \sum_i f_i(x,t)$$

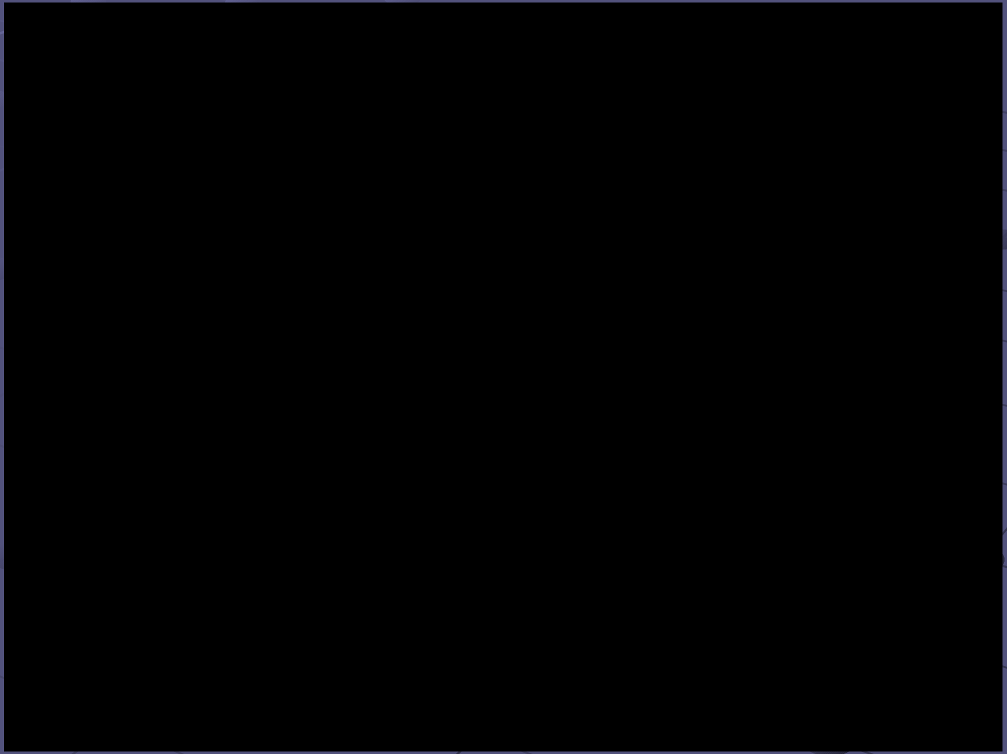
$$u(x,t) = \frac{1}{\rho(x,t)} \sum_i f_i(x,t)\, c_i$$

- Adding additional forces conceptually easy

$$u_{new}(x,t) = u(x,t) + \frac{\tau F}{\rho}$$

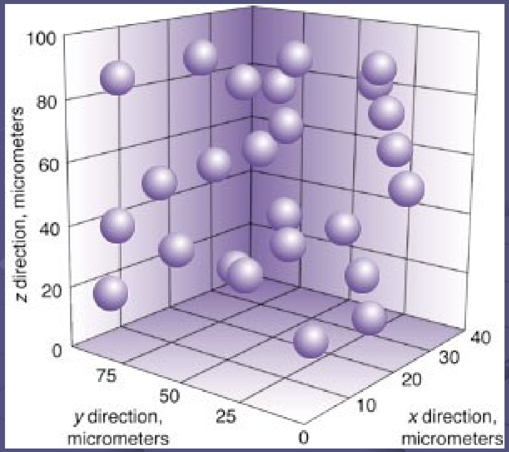
Lattice Boltzmann Applications

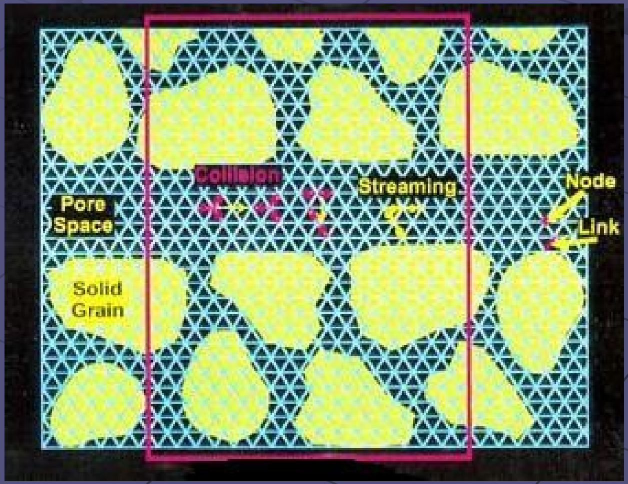
- Applied to many physical problems like shallow water equation, plasma physics
- Very successfull in simulating time-dependent fluid flow problems
- Multicomponent fluid problems been studied as well
- Ideal for parallelization



Particle Lattice Boltzmann Method

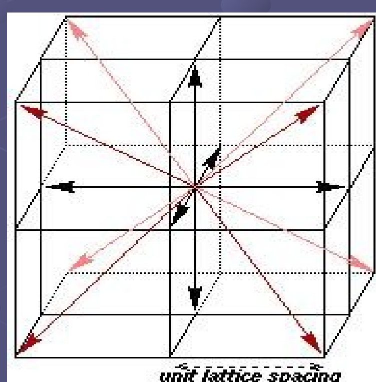
- Particle based nature simplifies simulation of particle suspension
- Reverse populations on solid wall boundaries
- Simulation of fluid flow in porous media is accomplished as well
- LBM in the study of fluid-structure interaction



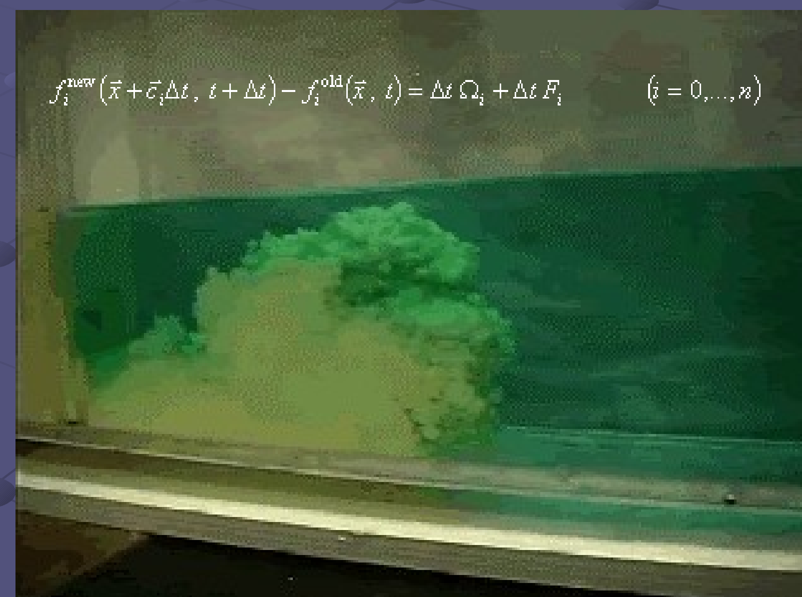


Turbidity Current Simulation with the Lattice-Boltzmann Method

- Relatively easy to incorporate sand-fluid interaction at the particle level
- Parameters have simple physical interpretations
- Turbulence modeling essential
- Work in progress



Simulation lattice: D3Q15



Conlusion

- Turbidity flow is a very comlex system to model
- Particle-based modeling possible alternate approach to conventional methods
- Lattice Boltzmann method is a good candidate for simulating turbidity flow
- More details in the article